LBL-36548
UCB-PTH-94/36
December 1994
hep-th/9412125

PAULI-VILLARS REGULARIZATION OF GLOBALLY SUPERSYMMETRIC THEORIES*

Mary K. Gaillard

Department of Physics and Theoretical Physics Group, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Abstract

It is shown that the one-loop ultraviolet divergences in renormalizable supersymmetric theories can be regulated by the introduction of heavy Pauli-Villars chiral supermultiplets, provided the generators of the gauge group are traceless in the matter representation. The procedure is extended to include supersymmetric gauged nonlinear sigma models.

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY–90–21139.

It was recently shown [1] that the one-loop quadratic divergences of standard supergravity can be regulated by the introduction of heavy Pauli-Villars fields belonging to chiral and abelian gauge multiplets. This result holds *a fortiori* for renormalizable supersymmetric theories. While it has not yet been shown that the full supergravity theory, including all logarithmic divergences, can be regulated in this way, the results of supergravity calculations [2], [3] can be used to show that this is true for the easier case of renormalizable supersymmetric theories, provided $\mathrm{Tr}(T_a)$ vanishes, where the T_a represent the generators of the gauge group on the light chiral multiplets. This result may be of some practical value in calculating quantum corrections, although there is no objection of principle to using dimensional regularization for these calculations. I also consider the class of nonrenormalizable theories with a nontrivial Kähler potential; in this case the Pauli-Villars masses, which play the role of effective cut-offs, acquire physical significance.

The Pauli-Villars regularization procedure is presented here using functional integration in an arbitrary bosonic background. That is, I calculate the one-loop correction to that part of the bosonic action that grows with the effective cut-off(s). Since the result just amounts to wave function renormalizations (and/or renormalization of the Kähler potential), the fermionic action can be inferred from supersymmetry. The derivative expansion used here to obtain the ultraviolet divergent part of the effective action is not generally amenable to the calculation of finite S-matrix matrix elements because higher order terms in the expansion are infrared divergent. However, since it is shown that the theory is ultraviolet finite (at least at one loop) with the appropriate choice of Pauli-Villars fields, there is no impediment in principle to implementing this regularization procedure in Feynman diagram calculations.

The one-loop effective action S_1 is obtained from the term quadratic in quantum fields when the Lagrangian is expanded about an arbitrary back-

ground; for a renormalizable gauge theory:

$$\begin{aligned}\mathcal{L}_{quad}(\Phi,\Theta,c) &= -\frac{1}{2}\Phi^T Z^\Phi\left(\mathcal{D}^2_\Phi + H_\Phi\right)\Phi + \frac{1}{2}\bar{\Theta}Z^\Theta\left(i\,\not{\mathcal{D}}_\Theta - M_\Theta\right)\Theta \\ &\quad +\frac{1}{2}\bar{c}Z^c\left(\mathcal{D}^2_c + H_c\right)c + O(\psi), \end{aligned} \tag{1}$$

where $\mathcal{D}_\mu$ is the gauge covariant derivative, the column vectors Φ,Θ,c represent quantum bosons, fermions and ghost fields, respectively, and ψ represents background fermions that are set to zero throughout this paper. The one loop bosonic action is given by

$$\begin{aligned} S_1 &= \frac{i}{2}\mathrm{Tr}\ln\left(\mathcal{D}^2_\Phi + H_\Phi\right) - \frac{i}{2}\mathrm{Tr}\ln\left(-i\,\not{\mathcal{D}}_\Theta + M_\Theta\right) + \frac{i}{2}\mathrm{STr}\ln\left(\mathcal{D}^2_c + H_c\right) \\ &= \frac{i}{2}\mathrm{STr}\ln\left(\mathcal{D}^2 + H\right) + T_-, \end{aligned} \tag{2}$$

where T_- is the helicity-odd fermion contribution which is finite for a renormalizable theory (and, more generally, in the absence of a dilaton [3]), and the helicity-even contribution is determined by

$$\mathcal{D}^2_\Theta + H_\Theta \equiv \left(-i\,\not{\mathcal{D}}_\Theta + M_\Theta\right)\left(i\,\not{\mathcal{D}}_\Theta + M_\Theta\right). \tag{3}$$

The field-dependent matrices $H(\phi)$ and $\mathcal{D}_\mu(\phi)$, as extracted by taking the flat (Kähler and space-time) limit of the results of [2], [3], are given below. Explicitly evaluating (2) with an ultraviolet cut-off Λ and a massive Pauli-Villars chiral supermultiplet sector with a squared mass matrix of the form

$$M^2_{PV} = H^{PV}(\phi) + \begin{pmatrix}\mu^2 & \nu \\ \nu^\dagger & \mu^2\end{pmatrix} \equiv H^{PV} + \mu^2 + \nu, \quad |\nu|^2 \sim \mu^2 \gg H^{PV} \sim H,$$

gives, with $H' = H + H^{PV}$:

$$\begin{aligned} 32\pi^2 S_1 &= -\int d^4x d^4p\,\mathrm{STr}\ln\left(p^2 + \mu^2 + H' + \nu\right) + 32\pi^2\left(S_1' + T_-\right) \\ &= 32\pi^2\left(S_1' + T_-\right) - \int d^4x d^4p\,\mathrm{STr}\ln\left(p^2 + \mu^2\right) \\ &\quad - \int d^4x d^4p\,\mathrm{STr}\ln\left[1 + \left(p^2 + \mu^2\right)^{-1}\left(H' + \nu\right)\right], \end{aligned} \tag{4}$$

where S'_1 is a logarithmically divergent contribution that involves the Yang-Mills field strength, $G_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu]$:

$$32\pi^2 S'_1 = \frac{1}{12}\int d^4x d^4p \mathrm{STr}\frac{1}{(p^2+\mu^2)}G'_{\mu\nu}\frac{1}{(p^2+\mu^2)}G'^{\mu\nu}, \quad G'_{\mu\nu} = G_{\mu\nu} + G^{PV}_{\mu\nu}. \tag{5}$$

Finiteness of (4) requires

$$\mathrm{STr}\mu^{2n} = \mathrm{STr}H' = \mathrm{STr}\left(2\mu^2 H' + \nu^2\right) = \mathrm{STr}H'^2 + \frac{1}{6}\mathrm{STr}G'^2 = 0. \tag{6}$$

The vanishing of $\mathrm{STr}\mu^{2n}$ is automatically assured by supersymmetry. Once the remaining conditions are satisfied we obtain

$$S_1 = -\int \frac{d^4x}{64\pi^2}\mathrm{STr}\left[\left(2\mu^2 H' + \nu^2 + H'^2 + \frac{1}{6}G'_{\mu\nu}G'^{\mu\nu}\right)\ln\mu^2\right] + \text{finite terms}. \tag{7}$$

The Lagrangian for light fields is

$$\begin{aligned}\mathcal{L} \;&=\; \mathcal{D}^\mu z^i \mathcal{D}_\mu \bar{z}_i - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - W_i\overline{W}^i - \frac{g^2}{2}\left(\bar{z}T^a z\right)\left(\bar{z}T_a z\right)\\ &\quad + \frac{i}{2}\left(\bar{\lambda}\,\not{\mathcal{D}}\lambda + \bar{\chi}^L_i\,\not{\mathcal{D}}\chi^i_L + \bar{\chi}^i_R\,\not{\mathcal{D}}\chi^R_i\right)\\ &\quad + \sqrt{2}g\left(i\bar{\lambda}^a_R(T_a\bar{z})_i\chi^i_L + \mathrm{h.c.}\right) + \mathcal{L}_{gf} + \mathcal{L}_{gh},\end{aligned} \tag{8}$$

where $Z^i = (z^i, \chi^i_L)$ is a chiral supermultiplet, λ^a is a gaugino, $F^a_{\mu\nu}$ is the Yang-Mills field strength, and $W(z)$ is the superpotential. The gauge-fixing and ghost terms [2] are, respectively,

$$\begin{aligned}\mathcal{L}_{gf} \;&=\; -\frac{1}{2}C_a C^a, \quad C^a = \mathcal{D}^\mu \hat{A}^a_\mu + ig\left[(\bar{z}T^a\hat{z}) - (\hat{\bar{z}}T^a\bar{z})\right],\\ \mathcal{L}_{gh} \;&=\; \frac{1}{2}\bar{c}_a\left[(\mathcal{D}_\mu\mathcal{D}^\mu)^a_b + g^2(\bar{z}T^a z)(\bar{z}T_b z) + g^2(\bar{z}T_b z)(\bar{z}T^a z)\right]c^b,\end{aligned} \tag{9}$$

where quantum variables are hatted and background fields are unhatted.

To regulate the theory we introduce Pauli-Villars regulator chiral supermultiplets ϕ_{PV}: $Z^I_\alpha = (\bar{Z}^\alpha_{\bar{I}})^\dagger$, $Z'^I_\alpha = (\bar{Z}'^\alpha_{\bar{I}})^\dagger$ and $\varphi^a_\beta = (\bar{\varphi}^\beta_a)^\dagger$, with signature $\eta^{\alpha,\beta} = \pm 1$, which determines the sign of the corresponding contribution

to the supertrace relative to an ordinary particle of the same spin. Thus $\eta = +1(-1)$ for ordinary particles (ghosts). Z^I transforms like z^i under the gauge group, Z'^I transforms according to the representation conjugate to z^i, and φ^a transforms according to the adjoint representation. Including these fields the superpotential is

$$\begin{aligned} W(Z^i, Z^I, Z'^I, \varphi^a) &= W(Z) + \sum_{\alpha,I} \mu_I^\alpha Z_\alpha^I Z_\alpha'^I + \sum_{a,\beta} \mu_\beta \varphi_a^\beta \varphi_\beta^a \\ &+ \frac{1}{2} \sum_\alpha a^\alpha W_{ij} Z_\alpha^I Z_\alpha^J + g^2 \sum_\gamma b^\gamma \theta_\gamma^a (Z'_\gamma T_a Z), \quad (10) \end{aligned}$$

where the ranges of summation are

$$\alpha = 1, \cdots, N_I, \quad \beta = 1, \cdots, N_\varphi, \quad \gamma = 1, \cdots, N_{I\varphi}, \quad N_{I\varphi} \leq \min\{N_I, N_\varphi\}, \quad (11)$$

The parameters μ play the role of effective cut-offs, and a^α, b^γ are of order unity.

The supertraces needed to evaluate the divergent part of the one-loop bosonic action are [2]–[4]

$$\mathrm{STr} H_\chi = 2g^2 (\bar{z} T_a z) \mathrm{Tr}(T^a), \quad \mathrm{STr} H_g = 0, \quad (12)$$

where the subscripts χ, g refer to supertraces over chiral and Yang-Mills (including ghosts) supermultiplets, respectively. Since the heavy Pauli-Villars fields form vector-like representations, they do not contribute to $\mathrm{Tr}(T_a)$; thus we require $\mathrm{Tr}(T_a) = 0$ for the light fields.

In addition, defining

$$X_A Y^A = \sum_A \eta^A X_A Y^A \quad (13)$$

we have, for traces evaluated with $\phi_{PV} = 0$:

$$\begin{aligned} \frac{1}{2} \mathrm{Tr} H_\chi^2 &= W_{iAB} \overline{W}^{ABj} \left(\overline{W}^i W_j + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}_j \right) \\ &- g^2 (\bar{z} T^a z) \left[(T_a z)^i W_{iAB} \overline{W}^{AB} - 4g^2 \left(\bar{z} C_2(R_z) T_a z \right) \right] \end{aligned}$$

$$
\begin{aligned}
&\quad -2g^4 C_G^a(\bar{z}T_a z)(\bar{z}T^a z) - \frac{g^2}{2}C_\chi^a\left[F_a^{\mu\nu}F_{\mu\nu}^a - 2g^2(\bar{z}T_a z)(\bar{z}T^a z)\right], \\
\frac{1}{2}\mathrm{STr}H_{\chi g}^2 &= -4g^2\left(\mathcal{D}_\mu\bar{z}C_2(R_z)T_a\mathcal{D}^\mu z\right) - 4g^2(T_a z)^i(T^a\bar{z})^{\bar{n}}W_{ki}\overline{W}_{\bar{n}}^k \\
\frac{1}{2}H_g^2 &= g^2 C_G^a\left[\frac{3}{2}F_{\mu\nu}^a F_a^{\mu\nu} - g^2(\bar{z}T_a z)(\bar{z}T^a z)\right], \\
\mathrm{STr}G_{\mu\nu}^\chi G_\chi^{\mu\nu} &= \mathrm{STr}G_{\mu\nu}^g G_g^{\mu\nu} = 0. \qquad (14)
\end{aligned}
$$

In these expressions, A, B refer to all (light and Pauli-Villars) chiral multiplets, C_G^a is the Casimir in the adjoint representation of the gauge subgroup G_a, $C_2(R)$ is the Casimir in the chiral multiplet representation R: $(\bar{z}T^a T_a z) = (\bar{z}C_2(R_z)z)$, and C_χ^a is defined by

$$
\begin{aligned}
C_\chi^a\delta_{ab} &= [\mathrm{Tr}(T_a T_b)]_{\text{light+PV}}, \quad C_\chi^a = \sum_{R_i} C_{R_i}^a\left(1 + 2\sum_\alpha \eta_I^\alpha\right) + C_G^a\sum_\beta \eta_a^\beta, \\
C_R^a\delta_{ab} &= [\mathrm{Tr}(T_a T_b)]_R, \quad C_R^a = \frac{\dim(R)}{\dim(G_a)}C_2^a(R), \quad C_2(R) = \sum_a C_2^a(R). \qquad (15)
\end{aligned}
$$

We obtain for the overall supertraces:

$$
\begin{aligned}
\frac{1}{2}\mathrm{STr}H &= \left[W_{iAB}\overline{W}^{ABj} - 4g^2\delta_i^j C_2(R_i)\right]\left(\overline{W}^i W_j + \mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}_j\right) \\
&\quad -g^2(\bar{z}T^a z)\left[(T_a z)^i W_{iAB}\overline{W}^{AB} - 4g^2\left(\bar{z}C_2(R_z)T_a z\right)\right] \\
&\quad +\frac{g^2}{2}\left(3C_G^a - C_\chi^a\right)\left[F_a^{\mu\nu}F_{\mu\nu}^a - 2g^2(\bar{z}T_a z)(\bar{z}T^a z)\right], \\
\mathrm{STr}G_{\mu\nu}G^{\mu\nu} &= 0, \qquad (16)
\end{aligned}
$$

where I used the identities

$$
(T^a z)^i W_i = 0 = \partial_j\left[(T^a z)^i W_i\right] = (T^a)^i_j W_j + (T^a z)^i W_{ij}, \qquad (17)
$$

that follow from the gauge invariance of the superpotential. Explicitly evaluating the derivatives of the superpotential with respect to the Pauli-Villars fields gives:

$$
\begin{aligned}
(T_a z)^i W_{iAB}\overline{W}^{AB} &= (T_a z)^i W_{ijk}\overline{W}^{jk}\left(1 + \sum_\alpha \eta_I^\alpha a_\alpha^2\right) + g^2\left(\bar{z}C_2(R_z)T_a z\right)\sum_\gamma \eta^\gamma b_\gamma^2, \\
W_{iAB}\overline{W}^{ABj} &= W_{ik\ell}\overline{W}^{jk\ell}\left(1 + \sum_\alpha \eta_I^\alpha a_\alpha^2\right) + g^2\delta_i^j C_2(R_i)\sum_\gamma \eta^\gamma b_\gamma^2. \qquad (18)
\end{aligned}
$$

Therefore finiteness requires:

$$0 = 1 + 2\sum_\alpha \eta_I^\alpha = 3 - \sum_\gamma \eta_a^\gamma = 1 + \sum_\alpha \eta_I^\alpha a_\alpha^2 = 4 - \sum_\gamma \eta_a^\gamma b_\gamma^2. \tag{19}$$

To extract the residual finite part, we may take, for example,

$$\eta_I^\alpha = \eta^\alpha, \quad \mu_I^\alpha \;=\; \mu\delta^\alpha, \quad \mu^\beta = \mu\epsilon^\beta, \tag{20}$$

and use the results of [5] to evaluate the terms in (7) that grow with the effective cut-off μ. Since there are no terms of order μ^2 in (14), we need only:

$$\sum_\alpha \eta_\alpha \ln(\mu^2\lambda_\alpha) = \ln\mu^2 \sum_\alpha \eta^\alpha + \ln\rho, \quad \ln\rho = \sum_\alpha \eta^\alpha \ln\lambda_\alpha. \tag{21}$$

Then we obtain for the one-loop effective action:

$$\begin{aligned}\mathcal{L}_1 \;&=\; -\frac{1}{64\pi^2}\mathrm{STr}\left(H'^2\ln\mu^2\right) + \text{finite terms}\\ &=\; \frac{1}{64\pi^2}\Bigg\{\left[\ln(\mu^2/\rho'_Z m'^2_Z)W_{ik\ell}\overline{W}^{jk\ell} - 4g^2\delta_i^j C_2(R_i)\ln(\mu^2\rho_{Z\varphi}/m^2_{Z\varphi})\right]\left(\overline{W}^i W_j + \mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}_j\right)\\ &\quad -g^2(\bar{z}T^a z)\left[\ln(\mu^2/\rho'_Z m'^2_z)(T_a z)^i W_{ijk}\overline{W}^{jk} - 4g^2\ln(\mu^2/\rho_{Z\varphi}m^2_{Z\varphi})\left(\bar{z}C_2(R_z)T_a z\right)\right]\\ &\quad +\frac{g^2}{2}\left[3C_G^a\ln(\mu^2\rho_\varphi/m^2_\varphi) - C_M^a\ln(\mu^2/\rho'_Z m'^2_Z)\right]\left[F_a^{\mu\nu}F^a_{\mu\nu} - 2g^2(\bar{z}T_a z)(\bar{z}T^a z)\right]\Bigg\}\\ &\quad +\text{finite terms.}\end{aligned} \tag{22}$$

The m's are the appropriate infrared cut-offs, and

$$\begin{aligned}\ln\rho_Z \;&=\; 2\sum_\alpha \eta^\alpha \ln\delta_\alpha^2, \quad \ln\rho_\varphi = \sum_\beta \eta^\beta \ln\epsilon_\beta^2,\\ \ln\rho'_Z \;&=\; \sum_\alpha \eta^\alpha \ln(\delta_\alpha a_\alpha)^2, \quad \ln\rho_{Z\varphi} = \sum_\gamma \eta^\gamma \ln(\delta_\gamma\epsilon_\gamma b_\gamma^2).\end{aligned} \tag{23}$$

Then the one-loop corrected bosonic Lagrangian can be written as

$$\mathcal{L}_0(z, A_\mu, g) + \mathcal{L}_1 = \mathcal{L}_0\left(Z_2^{-\frac{1}{2}}z, Z_3^{-\frac{1}{2}}A_\mu, Z_3^{\frac{1}{2}}g\right) + \text{finite terms}, \tag{24}$$

with

$$\left(Z_2^{-1}\right)_i^j = \frac{1}{32\pi^2}\left[\ln(\mu^2/\rho'_Z m'^2_Z)W_{ik\ell}\overline{W}^{jk\ell} - 4g^2\delta_i^j \ln\left(\mu^2\rho_{Z\varphi}/m^2_{Z\varphi}\right)\sum_a C_2^a(R_i)\right],$$

$$Z_3^a = \frac{g^2}{16\pi^2}\left[3C_G^a \ln(\mu^2\rho_\varphi/m^2_\varphi) - C_M^a \ln(\mu^2/\rho'_Z m'^2_Z)\right], \quad C_M^a = \sum_R C_R^a. \quad (25)$$

The coefficients of $\ln\mu^2$ in the renormalization constants correspond to the standard result for the SUSY β-function and to the chiral wave function renormalization found in "supersymmetric" gauges [6], [2] and in string loop calculations [7].

It is straightforward to extend these results to the nonlinear σ-model, again using the results of [2], [3] for supergravity. We introduce an arbitrary Kähler potential $K(Z^i, \bar{Z}^i)$, with Kähler metric $K_{i\bar{m}}$; as in [1], Z^I, Z'^I have the same Kähler metric as Z^i, while φ^a has Kähler metric δ_{ab}. However to regulate the logarithmic divergences, we have to specify additional terms in K. With the conventions that indices are raised and lowered with the Kähler metric, and that scalar derivatives are field reparameterization covariant:

$$\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}_i = \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}}, \quad \bar{Z}_I = K_{i\bar{m}}(z^j, \bar{z}^{\bar{j}})\bar{Z}^{\bar{M}},$$
$$W_{ij} = \partial_i W_j + \Gamma^k_{ij} W_k \quad \text{etc.}, \quad (26)$$

we take

$$K(Z^i, \bar{Z}^{\bar{\imath}}, \phi_{PV}) = K(Z^i, \bar{Z}^{\bar{\imath}}) + \sum_\alpha \left(Z^I_\alpha \bar{Z}^\alpha_I + Z'^I_\alpha \bar{Z}'^\alpha_I\right) + \sum_\beta \bar{\varphi}^\beta_a \varphi^a_\beta$$
$$+\frac{1}{2}\left[\sum_{\alpha;\, I,J=i,j} K_{ij}(z^k, \bar{z}^{\bar{k}})\left(Z^I_\alpha Z^J_\alpha + Z'^I_\alpha Z'^J_\alpha\right) + \text{h.c.}\right] + O\left(\phi^2_{PV}\right), \quad (27)$$

and, Eqs. (12) and (14) are modified as follows:

$$\mathrm{STr} H_\chi = 2g^2\mathcal{D}^a D_A(T_a z)^A - 2R^j_i\left(\overline{W}^i W_j + \mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}_j\right),$$
$$D_A(T_a z)^A = \mathrm{Tr}(T_a) + \Gamma^A_{Ai}(T_a z)^i = \mathrm{Tr}(T_a) + \left(1 + 2\sum_\alpha \eta^\alpha_I\right)\Gamma^A_{Ai}(T_a z)^i,$$
$$\mathcal{D}_a = K_i(T_a z)^i, \quad R^j_i = R^{j\ A}_{i\ A} = \left(1 + 2\sum_\alpha \eta^\alpha_I\right) R^{j\ k}_{i\ k}, \quad (28)$$

so $\mathrm{STr}H_\chi$ vanishes by the conditions (19) with $\mathrm{Tr}(T_a) = 0$. Here R^A_B and $R^{A\ C}_{\ B\ D}$ are the Kähler Ricci and Riemann tensors, respectively. In addition:

$$\begin{aligned}
\frac{1}{2}\mathrm{Tr}H^2_\chi &= \left(W_{iAB}\overline{W}^{ABj} + 2W_{AB}\overline{W}^{AC}R^{B\ j}_{\ C\ i} + W_k\overline{W}^\ell R^{A\ j}_{\ B\ \ell}R^{B\ k}_{\ A\ i}\right)\left(\overline{W}^i W_j + \mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}_j\right) \\
&\quad + 2\mathcal{D}_\mu z^j\mathcal{D}^\mu\bar{z}_i R^{i\ B}_{\ j\ A}R^{\ell\ A}_{\ k\ B}W_\ell\overline{W}^k \\
&\quad + \mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\left(W_{iAB}\overline{W}^k R^{A\ \ B}_{\ k\ \ j} - R^{A\ B}_{\ j\ i}W_{kAB}\overline{W}^k + \mathrm{h.c.}\right) \\
&\quad + \mathcal{D}_\mu z^j\mathcal{D}^\mu\bar{z}_i\mathcal{D}_\nu z^\ell\mathcal{D}^\nu\bar{z}_k\left(R^{A\ i}_{\ B\ j}R^{B\ \ k}_{\ A\ \ \ell} - \frac{1}{2}R^{A\ i}_{\ B\ \ell}R^{B\ \ k}_{\ A\ \ i}\right) \\
&\quad + \mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}_k\mathcal{D}^\nu\bar{z}_\ell\left(R^{A\ B}_{\ j\ i}R^{k\ \ell}_{\ A\ B} + \frac{1}{2}R^{A\ k}_{\ B\ i}R^{A\ j}_{\ B\ \ell}\right) \\
&\quad - g^2\mathcal{D}^a(T_a z)^i D_i\left[W_{AB}\overline{W}^{AB} - 4g^2(\bar{z}T^b)_j(T_b z)^j\right] - 2g^4C^a_G\mathcal{D}_a\mathcal{D}^a \\
&\quad - \frac{g^2}{2}D_A(T^b z)^B D_B(T_a z)^A\left(F^{\mu\nu}_b F^a_{\mu\nu} - 2g^2\mathcal{D}_a\mathcal{D}^a\right) \\
&\quad + 4g^2(\bar{z}T_b)_\ell(T^b z)^k\left[R^{\ell\ i}_{\ k\ j}W_i\overline{W}^j + 2g^2\mathcal{D}_a R^{A\ j}_{\ B\ i}W_j\overline{W}^i D_A(T^a z)^B\right] \\
&\quad + 2g^2\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}_j\left[2R^{j\ k}_{\ i\ \ell}(\bar{z}T^a)_k(T_a z)^\ell + \mathcal{D}_a R^{j\ A}_{\ i\ B}D_A(T^a z)^B\right] \\
&\quad + g^2\mathcal{D}_a(T^a z)^i R^{\ A\ \ B}_{i\ \ j}\overline{W}^j W_{AB} - 2iF^a_{\mu\nu}D_A(T_a z)^B R^{A\ \ j}_{\ B\ \ i}\mathcal{D}^\mu z^i\mathcal{D}^\nu\bar{z}_j, \\
\frac{1}{2}\mathrm{STr}H^2_{\chi g} &= -4g^2D^k(T_a\bar{z})_i\mathcal{D}_\mu\bar{z}_k D_j(T^a z)^i\mathcal{D}^\mu z^j - 4g^2(T_a z)^i(T^a\bar{z})^{\bar{n}}W_{ki}\overline{W}^k_{\bar{n}} \\
\frac{1}{2}H^2_g &= g^2C^a_G\left(\frac{3}{2}F^a_{\mu\nu}F^{\mu\nu}_a - g^2\mathcal{D}_a\mathcal{D}^a\right).
\end{aligned} \tag{29}$$

Using the conditions (19) we have

$$\begin{aligned}
D_A(T^b z)^B D_B(T_a z)^A &= D_i(T^b z)^j D_j(T_a z)^i\left(1 + 2\sum_\alpha\eta^\alpha_I\right) + \delta^b_a C^a_G\sum_\beta\eta^\beta_a = 3\delta^b_a C^a_G, \\
(T_a z)^i D_i W_{AB}\overline{W}^{AB} &= (T_a z)^i W_{ijk}\overline{W}^{jk}\left(1 + \sum_\alpha\eta^\alpha_I a^2_\alpha\right) \\
&\quad + g^2(\bar{z}T^b)(T_a z)^i D_i(T_b z)^j\sum_\gamma\eta^\gamma_a b^2_\gamma + O(\mu^2) \\
&= 4g^2(\bar{z}T^b)(T_a z)^i D_i(T_b z)^j + O(\mu^2), \\
W_{iAB}\overline{W}^{ABj} &= W_{ik\ell}\overline{W}^{jk\ell}\left(1 + \sum_\alpha\eta^\alpha_I a^2_\alpha\right) \\
&\quad + g^2D_i(T_b z)^k D^j(\bar{z}T_b)_k\sum_\gamma\eta^\gamma_a b^2_\gamma + O(\mu^2)
\end{aligned}$$

$$
\begin{aligned}
&= 4g^2 D_i(T_b z)^k D^j(\bar{z}T_b)_k + O(\mu^2) \\
W_{AB}\overline{W}^{AC} R_{C\,i}^{B\,j} &= W_{k\ell}\overline{W}^{km} R^{\ell\,j}_{m\,i}\left(1 + 2\sum_\alpha \eta_I^\alpha\right) \\
&\quad - g^2(\bar{z}T_a z)_\ell (T^a z)^k R^{\ell\,j}_{k\,i}\sum_\gamma \eta_a^\gamma b_\gamma^2 + O(\mu^2) \\
&= -4g^2(\bar{z}T_a z)_\ell (T^a z)^k R^{\ell\,j}_{k\,i} + O(\mu^2), \\
R^{A\,j}_{B\,\ell} R^{B\,k}_{A\,i} &= R^{m\,j}_{n\,\ell} R^{n\,k}_{m\,i}\left(1 + 2\sum_\alpha \eta_I^\alpha\right) = 0, \\
W_{iAB} R^{A\ \,B}_{\ k\ \,j} &= W_{i\ell m} R^{\ell\ \,m}_{\ k\ \,j}\left(1 + \sum_\gamma \eta^\gamma a_\gamma^2\right) = 0, \\
R^{A\,i}_{B\,\ell} R^{B\ \,k}_{\ A\ \,i} &= R^{j\ \,i}_{\ m\ \,\ell} R^{m\,k}_{\ j\ \,i}\left(1 + 2\sum_\alpha \eta_I^\alpha\right) = 0, \\
R^{A\,B}_{\ j\ \,i} R^{k\,\ell}_{\ A\,B} &= R^{k\,m}_{\ j\ \,i} R^{k\,\ell}_{\ k\,m}\left(1 + 2\sum_\alpha \eta_I^\alpha\right) = 0, \\
R^{A\,j}_{B\,i} D_A(T^a z)^B &= R^{k\,j}_{\ell\ \,i} D_k(T^a z)^\ell\left(1 + 2\sum_\alpha \eta_I^\alpha\right) = 0.
\end{aligned}
\tag{30}
$$

Inserting these results in (29), we see that the overall supertrace is finite. The $O(\mu^2)$ terms in (30) are the contributions from $Z^I, \bar{Z}'^I$ given in [1], and they contribute an $O(\mu^2)$ correction to the Kähler potential that was evaluated in that paper.

The residual finite terms that grow as $\ln\mu^2$ can be evaluated using (21); as shown in [2], in the absence of gauge couplings, they can be absorbed into a redefinition of the superpotential, except for some terms that depend the Kähler Riemann tensor[1], and that correspond to higher dimension superfield operators. When the gauge couplings are included [3], there is an additional correction to the Kähler potential, giving:

$$
\delta K(z,\bar{z}) = \frac{1}{32\pi^2}\left[\ln(\mu^2/\rho'_Z m'^2_Z) W_{ij}\overline{W}^{ij} - 4g^2 \ln\left(\mu^2 \rho_{Z\varphi}/m^2_{Z\varphi}\right)(\bar{z}T_a)_i (T^a z)^i\right]. \tag{31}
$$

[1]The next to last line of Eq.(3.6) of [2] should be deleted; other corrections to [2], that do not affect the final result presented in that equation, are given in [3].

The bosonic part of the correction to the couplings of the Yang-Mills superfields takes the form:

$$\delta\mathcal{L}_{gauge} = -\frac{g^2}{16\pi^2}\left(\frac{1}{4}F_b^{\mu\nu}F^a_{\mu\nu} - \frac{g^2}{2}\mathcal{D}_a\mathcal{D}^a\right)\Bigg[3C_G^a\delta_b^a\ln(\mu^2\rho_\varphi/m_\varphi^2) - \ln(\mu^2/\rho'_Z m'^2_Z)D_i(T^b z)^j D_j(T_a z)^i\Bigg]. \tag{32}$$

Aside from the holomorphic anomaly [8] that arises from the field-dependence of the infrared regulator masses, the coefficient of $F^{\mu\nu}F_{\mu\nu}$ is not a holomorphic function. That is, when the Kähler metric is not flat, there are corrections that correspond to D-terms as well as the usual F-terms.

It was shown in [1], in the context of supergravity, that Pauli-Villars regularization of the quadratic divergencies is still possible when the theory includes a dilaton supermultiplet coupled to the Yang-Mills supermultiplet; the regularization of all one-loop ultraviolet divergences when a dilaton is present will be considered elsewhere.

Acknowledgements. I thank Hitoshi Murayama for his interest in this work. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-WC03-76SF00098 and in part by the National Science Foundation under grant PHY-90-21139.

References

[1] M.K. Gaillard, Berkeley preprint LBL-36051/UCB-PTH-94/23, to be published in *Phys. Lett.* **B**.

[2] M.K. Gaillard and V. Jain, *Phys. Rev.* **D49:** 1951 (1994).

[3] M.K. Gaillard, V. Jain and K. Saririan Berkeley-Stony Brook preprint LBL-34948/UCB-PTH-93/37/ITP-SB-93-89 (1994).

[4] J. W. Burton, M.K. Gaillard and V. Jain, *Phys. Rev.* **D41:** 3118 (1990).

[5] P. Binétruy and M.K. Gaillard, *Nucl. Phys.* **B312:** 341 (1989); see Appendix C.

[6] R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo and C.A. Savoy, *Phys. Lett.* **115B:** 212 (1982).

[7] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, Northeastern U. preprint NUB-3057 (1992).

[8] L. Dixon, V. Kaplunovsky and J. Louis, *Nucl. Phys.* **B355** 649 (1991).